Motion along curved path pohyb v zatáčce

DOI: 10.46932/sfjdv3n4-001

Received in: April 14th, 2022
Accepted in: June 30th, 2022

Mustafa Osman Elrayah Aboelhassan
PhD in Electrical & Electronic Engineering, Electrical drives and Power Electronics.
Institution: University of Nottingham & Brno Univereity of Technology
Address: London, Greenwich, UK.
E-mail: mustafa.elrayah1@gmail.com

ABSTRACT
We all experience motion along a curved path in our daily life. The motion is not exactly a circular motion. However, we can think of curved path as a sequence of circular motions of different radii. As such, motion of vehicles like that of car, truck etc, on a curved road can be analyzed in terms of the dynamics of circular motion. Clearly, analysis is done for the circulr segment with the smallest radius as it represents the maximum curvature. It must be noted that „curvature" and „radius of curvature" are inverse to each other.

Keywords: motion, circular motion, curved path, vertical curves, curvature.

1 INTRODUCTION

If someone is sitting in the middle of the back seat, he/she holds on the fixed prop to keep the posture steady and move along with the motion of the car. If the person is close to the further side (from the center of motion) of the car, then she/ he leans to the side of the car to become part of the motion of the car. In either of the two situations, the requirement of the centripetal force for circular motion is fulfilled. The bottom of the body is in contact with the car and moves with it, whereas the upper part of the body is not. However, the side of the car applies normal force when the person leans on the side of the car away from the center of motion. Applied normal force meets the requirement of centripetal force. Finally, once the requirement of centripetal force is met, the whole body is in motion with the car. The body seeks to move straight, but the lower part in contact with car moves along curved path having side way component of motion (towards the side). The result is that the upper part is away from the center of curvature of the curved path. In order to keep the body upright, an external force in the radial direction is required to be applied on the body.

Let us concentrate on what is done to turn a car to negotiate a sharp turn. The driver of a car simply guides the steering of the car to move it along the turn. Intuitively, we think that the car engine must be responsible for meeting the requirement of centripetal force. This is right. However, engine does not directly apply force to meet the requirement of centripetal force. It is actually the friction resulting from the motion caused by engine, which acts towards the center of circular path and meets the requirement of centripetal force. The body of the car tends to move straight in accordance with its natural tendency. As the wheel is made to move side way (by the change in direction), the wheel has the tendency to have relative motion with respect to the road in the direction away from the center of path. In turn, road applies friction, which is directed towards the center.

2 MOTION ALONG CURVED PATH

Figure 1 Motion along curved path

Important to note here is that we are not considering friction in the forward or actual direction of motion, but perpendicular to actual direction (side way). There is no motion in the side way direction, if there is no side way skidding of the car. In that case, the friction is static friction (F_s) and has not exceeded maximum or limiting friction (f_s). Thus,

$$f_s \leq F_s$$
$$f_s \leq \mu_s N \quad (1)$$

There is no motion in vertical direction. Hence,

$$N = mg \quad (2)$$

Combining two equations, we have:

$$f_s \leq \mu_s Ng \quad (3)$$

where „m“ is the combined mass of the car and the passengers

In the limiting situation, when the car is about to skid away, the friction force is equal to the maximum static friction, meeting the requirement of centripetal force required for circular motion.

$$\mu_s mg = \frac{mv^2}{r}$$
$$v = \sqrt{\mu_s rg} \qquad (4)$$

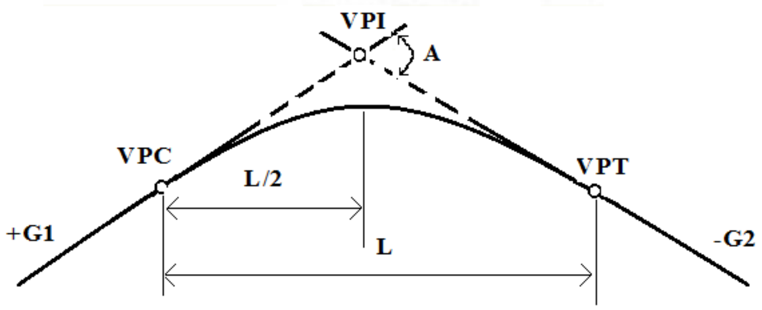
The following points can be noted about the condition as put on the speed of the car:

- There is a limiting or maximum speed of car to ensure that the car moves along curved path (*Figure 1*).
- If the limiting condition with regard to velocity is not met ($v = \sqrt{\mu_s rg}$), then the car will skid away.
- The limiting condition is independent of the mass of the car.
- If friction between tyres and the road is more, then we can negotiate a curved with higher speed and vice-versa. This explains why we drive slow on slippery road.
- The smaller the radius of curvature, the smaller the limiting speed. This explains why sharper turn (smaller radius of curvature) is negotiated with smaller speed.

3 VERTICAL CURVES

A vertical curve (*Figure 2*) is applied at an intersection of two slopes on a highway or a roadway to provide safe and comfort ride for vehicles on a roadway.

Figure 2 Vertical Curve



From the above figure,
VPC: Veritical Point of curvature
VPI: Veritical Point of Intersection
VPT: Veritical Point of Tangency
G1, G2: Tangent grades in percent
L: Length of vertical curved

Two types of vertical curves will be Clarified (*Figures 3 & 4*):

I. Crest Vertical Curves (**Type I and II**)

Minimum length of a crest vertical curve needs to satisfy the safety, comfort and appearance criteria. It is equal to three time the design speed. General equation for the length of a crest vertical curve (in terms of algebraic difference in grades) are often used to check the design speed of existing vertical curve.

when sight distance is less than length of vertical curve

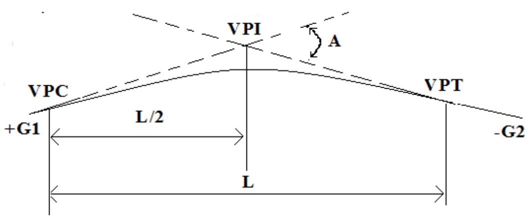
$$L = \frac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2} \tag{5}$$

When when sight distance is greater than length of vertical curve

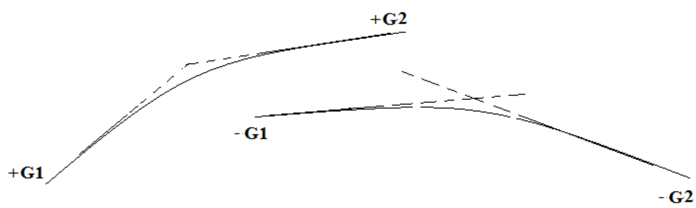
$$L = 2S - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{A} \tag{6}$$

where,
L = Length of vertical curve
S = Sight distance
A= Algebricdifference in grades, percent
h_1= Height of eye above roadway
h_2= Height of object above roadway surface

Figure3 Crest Vertical Curves



Types I



Types II

II. Sag Vertical Curves (**Type III and IV**)

A design of sag vertical curves need to satisfy at least four difference criteria:

1. General appearance
2. Drainage control
3. Passenger comfort
4. Head light sight distance (The design length of a sag vertical curve is based on the head light distance)

when sight distance is less than length of vertical curve

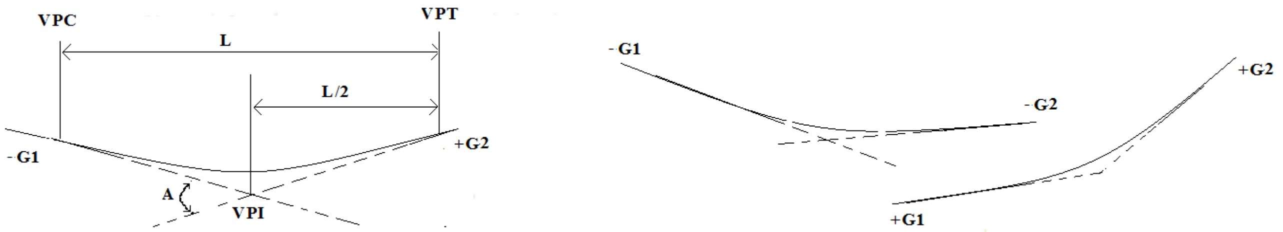
$$L = \frac{AS^2}{200(\sqrt{h_1} + S \tan \beta)} \tag{7}$$

When when sight distance is greater than length of vertical curve

$$L = 2S - \frac{200(h_1 + S \tan \beta)}{A} \tag{8}$$

where,
L = Length of sag vertical curve
S = Light beam disatance
A = Algebric difference in grades, percent
h_1 = Head light height
β = Angle of light beam intersects the surface of the roadway, degree

Figure 4 Sag Vertical Curves



Types III

Types IV

where,
G1 and G2 = Tangent grades in percent
A = Algebraic difference in grade
L = Length of vertical curve

The horizontal distance in feet (meters) needed to make one percent change in gradient to determine the minimum length of vertical curved. Additionally, to determine the horizontal distance from the VPC to the high point of type I or the low point of type III.

$$K = \frac{L}{A} \tag{9}$$

4 CONCLUSION

Motion along curved path has been described. A crest vertical curve is defined as one in which the algebraic difference between the intersecting gradients is positive. The safety concern relative to crest vertical curves to SSD (Stopping Sight Distance) the distance at which a driver can see an object in the road ahead. A sag vertical curve is defined as one in which the algebraic difference between the intersecting gradients is negative. The safety concern relative to sag vertical curves is related to HSD (Heading sight distance).

REFERENCE

[1] American Association of State Highway and Transportation Officials (AASHTO-2004). A policy on Geometric Design of Highways and Streets, Fifth Edition. Washington, D.C.

[2] Manual on uniform Traffic Control Devices (MUTCD-2003). Millennium Edition.