Experimental Analysis of Determination of Earth's Gravitational Acceleration using The Concept of Free-Fall Motion and Conservation of Mechanical Energy

Análisis experimental de la determinación de la aceleración gravitatoria de la Tierra utilizando el concepto de movimiento de caída libre y la conservación de la energía mecánica

DOI: 10.46932/sfjdv2n3-075

Received in: May 1st, 2021

Accepted in: Jun 30th, 2021

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ABSTRACT

Measurements of gravitational acceleration in this study use the concepts of free-fall motion and mechanical energy conservation. The purpose of this study is to compare the values of gravitational acceleration obtained from the two experiments and determine the factors causing deviations from the measurements of the gravitational acceleration value from free-fall motion experiments and the law of mechanical energy conservation. The research method used was a descriptive analysis of primary data in the Applied Physics Laboratory of Politeknik Negeri Bandung. The data was collected using free-fall motion equipment and the law of mechanical energy conservation. Height is the independent variable, and time is the dependent variable. The data were processed using a computer-aided device, and it is obtained the g value with the concept of free-fall and mechanical energy conservation by 9.54 m/s^2 and 10.1 m/s^2 . The deviation of g value in the free-fall motion is because of the presence of magnetic remanence that holds the ball from falling immediately when the time calculator was operated. The result, which is too large, causes the value of g too small. The result of g deviation in the mechanical energy conservation is caused by determining the location of the light source when the ball is in a stable condition. Meanwhile, in the real measurement, the ball moves as it passes through the light sensor, which results in a large tension force so that the sensor position becomes higher. This results Δt becomes smaller than the real one so that v becomes too large, and as a consequence, the value of g becomes too large.

RESUMEN

Las mediciones de la aceleración gravitacional en este estudio utilizan los conceptos de movimiento de caída libre y conservación de la energía mecánica. El propósito de este estudio es comparar los valores de la aceleración gravitacional obtenidos en los dos experimentos y determinar los factores que causan las desviaciones de las mediciones del valor de la aceleración gravitacional de los experimentos de movimiento de caída libre y la ley de conservación de la energía mecánica. El método de investigación utilizado fue un análisis descriptivo de datos primarios en el Laboratorio de Física Aplicada del Politeknik

Negeri Bandung. Los datos se recogieron utilizando el equipo de movimiento de caída libre y la ley de conservación de la energía mecánica. La altura es la variable independiente y el tiempo es la variable dependiente. Los datos se procesaron utilizando un dispositivo asistido por ordenador, y se obtiene el valor de g con el concepto de caída libre y conservación de la energía mecánica por 9,54 m/s² y 10,1 m/s². La desviación del valor de g en el movimiento de caída libre se debe a la presencia de remanencia magnética que impide que la bola caiga inmediatamente cuando se acciona la calculadora de tiempo. El resultado de t , que es demasiado grande, hace que el valor de g sea demasiado pequeño. El resultado de la desviación de g en la conservación de la energía mecánica es causado por la determinación de la ubicación de la fuente de luz cuando la bola está en una condición estable. Mientras tanto, en la medición real, la bola se mueve al pasar por el sensor de luz, lo que resulta en una gran fuerza de tensión para que la posición del sensor sea mayor. Esto resulta en que Δt se hace más pequeño que el real de modo que v se hace demasiado grande, y como consecuencia, el valor de g se hace demasiado grande.

1 INTRODUCTION

The most encountered force in daily life is the gravitational pull of the earth on an object. This force is called gravity. If an object is dropped on the earth's surface by ignoring air resistance, then the only force acting on the object is the force from the earth's gravity. One of the cases of motion involving the force of gravity is the motion when the foot lands on the ground on a volleyball. The landing moment is the most critical and damaging moment in the process, because we are generating a force of 2 to 3 times the normal body weight on the area of first contact with the ground[1].

An object is accelerated to the earth by the earth's gravitational acceleration (g) 9.81 m/s² [2]. At each point, this acceleration is the same for all objects, and it does not depend on its mass.

The formula of Newton's law of universal gravitation [3] is

$$F = G \frac{m_1 m_2}{r^2} \quad (1)$$

where

F = gravitational force acting between two objects, m_1 and m_2

m_1 = mass of object 1

m_2 = mass of object 2

r = distance between the centres of the masses

G = gravitational constant

This law applies to all materials in the universe. So, according to this law, the object weight on earth is

$$w = G \frac{Mm}{r^2} \quad (2)$$

where

M = mass of the earth

m = mass of an object

r = distance between an object and the centre of the earth

According to Newton's II law, $\sum F = ma$ where $w = mg$, then the gravitational acceleration is obtained

$$g = G \frac{M}{r^2} \quad (3)$$

From equation (3), careful measurement of the g value in various locations shows that the g value does not have the same value in each location. The earth's gravitational force of an object changes based on location. Specifically, at a specific point on the earth's surface, the force due to the earth's gravity changes inversely with the square of the object distance to the centre of the earth. The measurement of g value in Yogyakarta by using the simple swing concept is 9.6 m/s^2 with a single measurement method and 9.8 m/s^2 with a repeated measurement method [4]. The measurement of g value at the Physics Laboratory of Politeknik Negeri Bandung is different. The g value from the measurements using free-fall motion experiments is 9.19 m/s^2 and the g value from the measurement using physical pendulum experiments is 9.77 m/s^2 [5].

The advances in science and technology produce various electronic devices that assist in the development of teaching aids. Some essential electronic devices in the development of learning media include control devices such as microcontrollers, detector devices or sensors, display devices or LCDs, and actuator devices [6]. The development of free-fall motion teaching materials is significant for learning outcomes with microcontroller-based experiments [7]. That learning materials affect the students' conceptual understanding.

The average g value using the concept of free-fall motion with the assistance of Arduino-based properties is 10.2 m/s^2 [8]. The free-fall experiment is to determine the acceleration due to gravity, g , with an accuracy of about 1 part in 10^4 [9]. The experiment is also to expose students to critical thinking in collecting, selecting, and analyzing data, and interpreting the result. The development of the free-fall motion learning media has been significant to reduce misconceptions and improve students' understanding of these concepts [10].

From the description above, determining the gravitational acceleration (g) value in a particular location can be obtained using many different experiments. Therefore, the purpose of this study is to measure the gravitational acceleration using the concept of mechanical energy conservation (MEC) and to compare it with experiments using the concept of free-fall motion (FFM). Then an analysis of the factors that led to the deviation of the measurement results is also carried out.

The measurement of gravitational acceleration examined experimentally in this study is the measurement using the concept of free-fall motion and the mechanical energy conservation because these two concepts are the subjects needed to fulfil learning outcomes in several engineering departments at Politeknik Negeri Bandung.

According to the kinematics equation of two-dimensional straight motion, free-fall motion is one of the motions which experiences a quite significant constant acceleration, that is the acceleration of the earth's gravity, and if the object initially moves from a position of rest, without initial velocity ($v_0 = 0$), then free-fall motion will complete the equation

$$h = \frac{1}{2}gt^2 \quad (4)$$

where h is height the object and t is travel time.

In the mechanical energy conservation experiment, when the steel ball is at height h , then it has relative potential energy of $U = mgh$ and when it is at a surface, zero height (is a reference to the calculation of potential energy, at this point the value is zero), then the potential energy has changed entirely into kinetic energy, $K = \frac{1}{2}mv^2$, assuming that there is no force acting on the steel ball other than the force of gravity, so the law of mechanical energy conservation applies. Therefore, the mechanical energy (E_{mec}) in the height h position is the same as the mechanical energy (E_{mec}) in the zero height position.

$$E_{mec_h} = E_{mec_0} \quad (5)$$

$$U = K \quad (6)$$

$$mgh = \frac{1}{2}mv^2 \quad (7)$$

$$v^2 = 2gh \quad (8)$$

Where v is velocity of the object at a surface or zero height position.

2 RESEARCH METHOD

The research method used was a descriptive analysis of primary data with the concept of free-fall motion and the law of mechanical energy conservation in the Applied Physics Laboratory of Politeknik Negeri Bandung. The data were collected using a free-fall motion experiment and mechanical energy

conservation device with height as an independent variable and time as a dependent variable. The data were processed using a computer-aided device.

Figure 1 presents the tools used for the free-fall motion experiment (a) and mechanical energy conservation law experiment (b).

FIGURE 1. Free-fall motion experiment device (a) and Mechanical Energy Conservation Experiment (b)

(a)

(b)

There are similarities in determining the value of gravitational acceleration using the FFM experiment to the one using the MEC experiment; they are releasing an object without initial velocity from a specific height and then making measurements that refer to the lowest point. In the FFM experiment, time is measured from start to finish, while in the MEC experiment, the speed at the final position is measured using the sensor's darkening time. The principal difference is the motion of the object. In the FFM experiment, the object takes a straight trajectory while in the MEC experiment, the object takes a circular trajectory. Therefore, in the MEC experiment, time is not measured from start to finish, but it makes use of the conservative concept of the gravitational field.

3 RESULT AND DISCUSSION

3.1 RESULTS OF COMPARISON BETWEEN TWO EXPERIMENTS

This study is measuring the earth's gravitational acceleration using the concepts of free-fall motion and mechanical energy conservation experiments.

In the free-fall motion experiment, Fig. 1 (a) shows a steel ball with a diameter of 30.0 mm held with a magnet. Shortly after the current is terminated, the steel ball will fall with constant acceleration because the gravitational force ($F = mg$) passes through the sensor, and the time calculation starts. In this case, air friction is ignored. When the steel ball hits the sensor, the time calculation by using the digital time calculator ends. Time (t) shows the time taken by the steel ball when falling from a specific height (h). Figure 2 presents the relation between height and time.

If equation (4) is converted to a linear equation $y = A + Bx$ with t^2 as abscissa (x) and h is as ordinate (y), then the gradient (B) = $\frac{1}{2}g$, so that $g = 2B$. Figure 2 presents the graph h of t^2 from the data of this study.

FIGURE 2. Graph h of t^2 for Free Fall Motion experiment

Figure 2 shows that the gradient value is 4.77 so that the g value = $2B = 2 \times 4.77 = 9.54 \text{ m/s}^2$. The graph also shows the curve has a point value with a negative y-axis of -0.035, which means that at the height of 0, the ball has time. This condition occurs because the magnet is too strong so that when the current has been terminated, the steel ball is still attached. Meanwhile, the enumerator has read the travel time, which causes the travel time becomes large. It results in a decrease in the g value. The relative error of the g value from the free-fall motion experiment is 2.9%. Thus this free-fall motion experiment can be used to determine the g value with good accuracy.

In the MEC experiment, Fig. 1 (b) shows a steel ball with 30.0 mm in diameter held by a massless rope and released from a specific height h . A light barrier is mounted at the base. The light barrier will read the time when the sensor is closed by the diameter of the ball. If the diameter of the ball is d and time when the light barrier is closed by diameter d is Δt then the speed of the ball when being dropped is $v = \frac{d}{\Delta t}$. The relation between these data, square of speed (v^2), and Fig. 3 shows the initial height (h).

Equation (8) is converted to a linear equation, the analogy with $y = A + BX$, with h as abscissa and v^2 as ordinate, so the gradient of $B = 2g$ or $g = B/2$.

FIGURE 3. Curve v^2 against h experiments of the Mechanical energy conservation

The curve in Fig. 3 shows the value of $B = 20.2$, the value of g is $20.2/2 = 10.1 \text{ m/s}^2$. This large value of g is due to the difficulty of placing the diameter of the ball right pass through the sensor. In other words, the sensor is not correctly covered by the ball diameter, causing time t read by the time calculator smaller, which results in the velocity of v as if it becomes large. According to equation (8), with large velocity, the value g becomes large. The relative error in determining the g value using the mechanical energy conservation experiment is 3.1%. Thus the mechanical energy conservation experiment can be used to determine the g value with reasonable accuracy.

In the experiments of free-fall motion and mechanical energy conservation, both have deviated g value average with almost the same deviation ($\approx 3\%$). The g value in the free-fall motion is smaller than the real g value, while in the mechanical energy conservation, the g value is larger than the real g value.

In the free-fall motion experiment, the formula $h = \frac{1}{2}gt^2$ or $g = \frac{2h}{t^2}$ is used. Because the measurement h is performed quite carefully, the uncertainty of g is mostly determined by the measurement of time, t . The measurement of measured t is greater than the real t because there is remanence of magnets holds the ball from falling immediately once the timer switch is operated. The too large t result causes too small g value.

In mechanical energy conservation experiment, a formula $v = \sqrt{2gh}$ or $g = \frac{v^2}{2h}$, where $v = \frac{d}{\Delta t}$ is used, where Δt is the darkening time of the light barrier sensor, and d is the diameter of the ball. The determination of light source location is performed when the ball is in a balanced position or a position of rest (Fig. 4 (a)). However, at the time of real measurement, the ball moves as it passes through the light sensor, which increases the tension force of the rope so that the sensor position becomes higher (Fig. 4 (b)).

FIGURE 4. Illustration of sensor position from the diameter of the ball

When checking the track, the ball is in a position of rest, as in Fig. 5(a),

$$T = mg \cos \theta \quad (9)$$

When measuring, the ball is in motion. From Newton's II law, the magnitude centripetal force the ball (Fig. 5 (b)) can be formulated as

FIGURE 5. Illustration of the steel ball as it passes through the light limiting sensor should be (a) and factual position (b)

$$\sum F_s = \frac{mv^2}{r} \quad (10)$$

$$T' - mg \cos \theta = \frac{mv^2}{r} \quad (11)$$

$$T' = mg \cos \theta + \frac{mv^2}{r} \quad (12)$$

(a)

(b)

So $T' > T$, because the rope is flexible, then the addition of the rope tension force results in the rope getting longer. This condition causes Δt smaller than the real one so that v becomes too large, and as a result, the g value becomes too large.

3.2 THE INFLUENCE OF AIR FRICTION

In the above discussion, the influence of air friction on objects, both for free-fall motion and mechanical energy conservation, is still ignored. This study discusses the influence of air friction in the free-fall motion experiment. If air friction should be taken into calculation, for free-fall motion object, working gravity force and viscous air friction, wherein the magnitude of friction is proportional to the square of velocity (v^2), but the direction is contrary to the velocity [11], [12], so the total force working on the object is:

$$F = mg - kv^2 \quad (13)$$

with viscous friction coefficient k , has a magnitude of

$$k = \frac{1}{2} \rho AC \quad (14)$$

Where ρ is the density of air, A is the cross section surface area of the object. In this case, the ball-shaped falling object, so that $A = \pi R^2$, with R stating the radius of the ball, and C is the drag coefficient for spherical objects worth empirically 0.47 [11]. By substituted that $F = m a = m (dv/dt)$, and $v = dh/dt$, where h is a vertical distance or height of falling object, then from equation (14), can be formulated a differential equation:

$$\frac{dv}{dt} + \left(\frac{k}{m}\right) v^2 = g \quad (15)$$

The solution of equation (15), in the form of relationship between distance (h) with travel time (t), is

$$h = \frac{m}{k} \ln \left[\cosh \left(t \sqrt{\frac{gk}{m}} \right) \right] \quad (16)$$

or

$$t = \sqrt{\frac{m}{gk}} \operatorname{acosh} \left[e^{\frac{hk}{m}} \right] \quad (17)$$

The solution to the relationship between distance and travel time with the correction of air friction is presented in Fig. 6. There is a difference between the time data on the calculation of air friction correction with time data of measurement results, where the measurement results are always higher than the result of air friction calculations. The difference is even more significant if travel time or distance is getting smaller. Thus, in addition to the air friction, other factors are involved slowing downtime of the object, which relates to the moment at the beginning of the object (the ball) is released from its holder right after the time calculator starts automatically. It is also clear from the Fig. 6. that at height $h=0$ means when the ball is released, time t is not zero. So there is a specific time necessary for the ball to start falling due to the magnetic remanence, as discussed previously.

FIGURE 6. The relationship between the distance and travel time squared with the correction of air friction

Figure 6 presents the difference between the measured travel time and the calculated results, where the measurement results are always larger than the calculation results. For the same distance, the measured travel time for a large ball is smaller than the measured travel time for small ball. However, the travel time, calculated using equation (8) for both small and large balls, is the same or nearly the same.

4 CONCLUSION

The free-fall motion and mechanical energy conservation experiments can be used to determine the value of the earth's gravitational acceleration. In the free-fall motion experiment and the mechanical energy conservation, both have the deviated average of g values with almost the same deviation. The g value in the free-fall motion experiment is smaller than the real g value, while in the experiment of mechanical energy conservation, the g value is larger than the real g value. The uncertainty of g in the free-fall motion is determined by measurement of time, t . The measurements of measured t are larger than

the actual one because there is remanence of magnets or magnets remaining that hold the ball from falling immediately once the time counter is operated. The result that is too large leads to the g value that is too small. This condition is further strengthened by the data when air friction comes into calculation. The uncertainty of g in the mechanical energy conservation experiment is caused by determining the location of the light source when the ball is in a balanced or a rest position. In the real measurement, the ball moves as it passes through the light sensor, which increases the tension force so that the sensor position becomes higher. As a result, Δt becomes smaller than the real one so that v becomes too large, and the g value becomes too large. The experiments of free-fall motion and the mechanical energy conservation show the consistency of conservative field conditions, even though the shapes of the object's trajectory in the two experiments are different. There is a difference in which one value is smaller than the actual value, while the other is bigger. This condition is due to the technical problems in measurement.

ACKNOWLEDGMENTS

This research has been funded by Politeknik Negeri Bandung through The Unit of Research and Community Service under the Individual Research Scheme year 2020, Contract Number B/249.114/PL1.R7/PG.00.03/2020 . The writers also thank all lecturers of Applied Physics of Politeknik Negeri Bandung for their contributions to this study.

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